WEEKLY TEST RANKER'S BATCH TEST - 02 RAJPUR
SOLUTION Date 08-09-2019

[PHYSICS]

1.

For elastic collision, $e = 1$. Let speed of the ball be v towards left after collision, then w.r.t wall, incident velocity = reflected velocity. We get $3 + 3 = v - 3 \Rightarrow v = 9$ m/s.

2.

Since, A and B have same mass. So, after elastic collision, they interchange their velocity.

$\therefore \quad v_B = v$

After collision between B and C,

$$v'_B = \left(\frac{m - 4m}{m + 4m}\right)v + \left(\frac{2 \times 4m \times 0}{m + 4m}\right) = -\frac{3}{5}v$$

Again, collision takes place between A and B. So, velocity will be inter changed.

$$\therefore \quad v'_A = (v'_B) = \frac{3}{5}v \text{ in left ward direction.}$$

3.

Let v_1, v_2 and v_3 be velocities of blocks 1, 2 and 3 after suffering collision each.

$$mv = mv_1 + Mv_2 \quad \text{and} \quad v_1 - v_2 = -v$$

Solving, we get $v_1 = \dfrac{m - M}{m + M} < 0$

$\therefore \quad m < M$

Fig. S7.24

$$\therefore \quad |v_1| = \frac{M - m}{m + M}v \qquad \text{(i)}$$

and $$v_2 = \frac{2m}{m + M} \times v_2 = \frac{4Mmv}{(m + M)^2} \qquad \text{(ii)}$$

$$\therefore \quad \frac{M - m}{M + m}v = \frac{4Mmv}{(M + m)^2}$$

$$M^2 - m^2 = 4Mm \Rightarrow \frac{M}{m} = 2 + \sqrt{5}$$

4.

The position of equilibrium corresponds to $F(x) = 0$

Since $F(x) = \frac{-dU(x)}{dx}$

so $F(x) = -\frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2}\right)$ or $F(x) = \frac{4a}{x^5} - \frac{2b}{x^3}$

For equilibrium, $F(x) = 0$, therefore

$$\frac{4a}{x^5} - \frac{2b}{x^3} = 0 \Rightarrow x = \pm\sqrt{\frac{2a}{b}}$$

$$\frac{d^2U(x)}{dx^2} = -\frac{20a}{x^6} + \frac{8b}{x^4}$$

Putting $x = \pm\sqrt{\frac{2a}{6}}$ gives $\frac{d^2U(x)}{dx^2}$ as negative

So U is maximum. Hence, it is position of unstable equilibrium

5.

Area between curve and displacement axis

$$= \frac{1}{2}\times(12+4)\times 10 = 80\ \text{J}$$

In this time, body acquires kinetic energy = $\frac{1}{2}mv^2$

By the law of conservation of energy

$$\frac{1}{2}mv^2 = 80\ \text{J}$$

$$\Rightarrow \quad \frac{1}{2}\times 0.1\times v^2 = 80$$

$$\Rightarrow \quad v^2 = 1600$$

$$\Rightarrow \quad v = 40\ \text{m/s}$$

6.

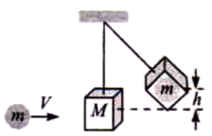
By the conservation of linear momentum

Initial momentum of sphere

= Final momentum of system

$mV = (m + M)v_{sys}$ ...(i)

If the system rises up to height h then by the conservation of energy



$$\frac{1}{2}(m+M)v_{\text{sys.}}^2 = (m+M)gh \quad \text{...(ii)}$$

$$\Rightarrow v_{\text{sys.}} = \sqrt{2gh}$$

Substituting this value in equation (i)

$$V = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

7.

As $m_1 = m_2$, therefore after elastic collision velocities of masses get interchanged
i.e. velocity of mass $m_1 = -5$ m/s
and velocity of mass $m_2 = +3$ m/s

8.

$$m_1 u_1 = m_2 v_2$$

$$\frac{1}{2} m_2 v_2^2 = \frac{1}{2}\left[\frac{1}{2} m_1 u_1^2\right]$$

$$\Rightarrow \quad (m_2 v_2) v_2 = \frac{1}{2}(m_1 u_1) u_1 \Rightarrow \quad v_2 = \frac{u_1}{2}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - 0}{u_1 - 0} = \frac{v_2}{u_1} \Rightarrow e = \frac{1}{2}$$

9.

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

For $m_1 << m_2$

$$m_1 - m_2 \simeq -m_2$$

$$m_1 + m_2 \simeq m_2$$

$$\Rightarrow \quad v_1 = -u_1 + 2u_2$$

$$\Rightarrow \quad v_1 = -12 + 2(10)$$

$$\Rightarrow \quad v_1 = 8 \text{ ms}^{-1}$$

10.

Tension in the cord is

$$T = M\left(g - \frac{g}{4}\right) = \frac{3Mg}{4}$$

in the upward direction. Since cord is displaced in the downward direction, so

$$W = \vec{T} \cdot \vec{d}$$

$$\Rightarrow \quad W = Td\cos(180)$$

$$\Rightarrow \quad W = -Td = -\frac{3Mgd}{4}$$

11.

TE = PE at height h is mgh

At height $3h/4$: $\text{PE} = mg\,\frac{3h}{4}$

$$\text{KE} = \text{TE} - \text{PE} = mg\,\frac{h}{4}$$

$$\therefore \quad \frac{\text{KE}}{\text{PE}} = \frac{1}{3} = 1:3.$$

12.

Applying the law of conservation of momentum,

$$mv = m_1\left(v_1 - \frac{v_1}{3}\right) \quad \text{or} \quad v = \frac{m_1}{m} \times \frac{2v_1}{3}$$

To describe a vertical circle v should be $\sqrt{5gl}$. Hence,

$$\sqrt{5gl} = \frac{m_1}{m} \times \frac{2v_1}{3} \quad \text{or} \quad v_1 = \left(\frac{m}{m_1}\right)\frac{3}{2}\sqrt{5gl}$$

13.

According to law of conservation of momentum,

$$mv + 0 = (m + M)V$$

or $$V = \left(\frac{mv}{m + M}\right)$$

As the block rises to height h, hence

$$V = \sqrt{2gh}$$

14.

Given that,

$\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ and $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

Hence, $v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$\therefore\ P = \vec{F}\cdot\vec{v} = 2t\cdot t^2 + 3t^2\cdot t^3 = 2t^3 + 3t^5$

15.

According to conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v,$$

where v is common velocity of the two bodies.

$m_1 = 0.1\,\text{kg}, m_2 = 0.4\,\text{kg}$

$v_1 = 1\,\text{m/s}, v_2 = -0.1\,\text{m/s}$

$\therefore\ 0.1\times1 + 0.4\times(-0.1) = (0.1 + 0.4)v$

or $0.1 - 0.04 = 0.5v,$

$$v = \frac{0.06}{0.5} = 0.12\ \text{m/s}.$$

Hence, distance covered $= 0.12 \times 10 = 1.2$ m

16.

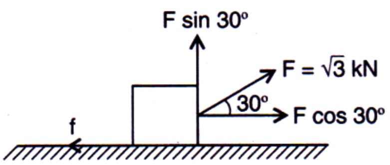
Height attained after n jumps

$$h' = e^{2n}h$$

As $n = 2$

Hence $h' = e^{2+2}h = e^4h$.

17.



The component of applied force F in the direction of motion is $F\cos30^\circ$.

The work done by the applied force is,

$$W = (F\cos30^\circ)S = \sqrt{3}\times10^3\times\frac{\sqrt{3}}{2}\times10\,\text{J}$$

$$= 15\times10^3\,\text{J} = 15\,\text{kJ}.$$

18.

19.

$$S = \frac{1}{3}t^2$$

$$v = \frac{dS}{dt} = \frac{2}{3}t;\qquad a = \frac{d^2S}{dt^2} = \frac{2}{3}$$

$$F = ma = 3\times\frac{2}{3} = 2\,\text{N};\quad \text{Work} = 2\times\frac{1}{3}t^2$$

At $t = 2$ seconds : Work $= 2\times\frac{1}{3}\times2\times2 = \frac{8}{3}$ J.

20.

Mass of neutron is, $m_1 = m$

Mass of alpha particle is, $m_2 = 4\ m$

Given: $u_1 = v,\ u_2 = 0$

The final velocity of the neutron after collision is given by:

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

$$= \frac{(m - 4m)v}{m + 4m} + \frac{2 \times 4m \times 0}{m + 4m} = -\frac{3v}{5}$$

21.

Force, $\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$ N

Displacement, $\vec{s} = 0\hat{i} + 10\hat{j} + 0\hat{k}$ m

$\therefore$work done $W = \vec{F} \cdot \vec{s}$

$$= (-2\hat{i} + 15\hat{j} + 6\hat{k})(0\hat{i} + 10\hat{j} + 0\hat{k}) = 150 \text{ J.}$$

22.

23.

24.

$mv_0 = (m + m)v$ or $v = v_0/2$

$$T = \frac{2mv^2}{l} + 2mg = \frac{2mv_0^2}{4l} + 2mg$$

$$= \frac{m(2gl)}{2l} + 2mg = 3mg$$

Initially, the tension $= T_0 = mg$

$\therefore$Increase in tension $= 2mg$.

25.

The time elapsed from the moment it is dropped to the second impact with the floor is,

$$t = \sqrt{\frac{2h}{g}}(1 + 2e)$$

where h is the initial height of the body from the ground

$$1.03 = \sqrt{\frac{2}{9.8}}(1 + 2e)$$

Solving, we get; $e = 0.64$

26.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$

$$2 \times 6 + 2 \times 0 = (2 + 2)V$$

or $V = 3 \text{ m s}^{-1}$

$$E = \frac{1}{2}(m_1 + m_2)V^2$$

$$= \frac{1}{2} \times 4 \times 9 = 18 \text{ J.}$$

27.

28.

Relative velocity of separation = relative velocity of approach

$= v$ (as $e = 1$)

$\therefore$ Time of next collision $= \dfrac{2\pi r}{v}$.

29.

TE = PE at height h is mgh

At height $3h/4$: PE $= mg\,\dfrac{3h}{4}$

KE = TE − PE $= mg\,\dfrac{h}{4}$

$\therefore \quad \dfrac{KE}{PE} = \dfrac{1}{3} = 1:3$.

30.

$$V = \frac{1}{2}kx^2$$

$$V' = \frac{1}{2}k(nx)^2 = n^2\left(\frac{1}{2}kx^2\right) = n^2V$$

31.

Acceleration $a = \dfrac{P}{m}$

Velocity after time t

$$v = at = \frac{Pt}{m}$$

Kinetic energy $= \dfrac{1}{2}m\left(\dfrac{Pt}{m}\right)^2 = \dfrac{P^2t^2}{2m}$

32.

33.

According to work-energy theorem

$(KE)_{final} - (KE)_{initial} = W$

$\therefore \quad (KE)_{final} - \dfrac{1}{2}\times 10\times 10^2 = \int_{20}^{30} F\,dx$

or $\quad (KE)_{final} = -\int_{20}^{30} \dfrac{x}{10}\,dx + 500$

$$= -\frac{1}{10}\left[\frac{x^2}{2}\right]_{20}^{30} + 500$$

$$= -\frac{1}{20}\left[(30)^2 - (20)^2\right] + 500$$

$$= -25 + 500 = 475 \text{ J}$$

34.

$$a = \left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right)g$$

Since, $v^2 - 0^2 = 2as$

$$\Rightarrow \quad v^2 = 2\left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right)gL$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right)gL}$$

35.

$$|F_c| = \frac{mv^2}{r} = \frac{k}{r^2}$$

$$mv^2 = \frac{k}{r}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = K.E. = \frac{k}{2r} \qquad ...(1)$$

Further

$$F = -\frac{dU}{dr}$$

$$\Rightarrow \quad -\frac{k}{r^2} = -\frac{dU}{dr}$$

$$\Rightarrow \quad dU = kr^{-2}dr$$

$$\Rightarrow \quad U = \int kr^{-2}dr$$

$$\Rightarrow \quad U = k\left[\frac{r^{-2+1}}{-2+1}\right] = -\frac{k}{r}$$

$\Rightarrow$ Total Energy, E is

$$E = K.E. + P.E.$$

$$\Rightarrow \quad E = \frac{k}{2r} - \frac{k}{r}$$

$$\Rightarrow \quad E = -\frac{k}{2r}$$

36.

$$dW = Fdx$$

$$\Rightarrow \quad W = \int_2^6 (5 + 3x)dx$$

$$\Rightarrow \quad W = 5(6-2) + \frac{3}{2}(6^2 - 2^2)$$

$$\Rightarrow \quad W = 20 + 48$$

$$\Rightarrow \quad W = 68 \text{ J}$$

37.

The initial and final kinetic energies are both zero, so the work done by the spring is the negative of the work done by friction, or $\frac{1}{2}kx^2 = \mu_k mgl,$ where l is the distance the block moves. Solving for μ_k,

$$\mu_k = \frac{(1/2)kx^2}{mgl} = \frac{(1/2)(100 \text{ N/m})(0.20 \text{ m})^2}{(0.50 \text{ kg})(10 \text{ m/s}^2)(1.00 \text{ m})} = 0.40$$

38.

If the body strikes the sand floor with a velocity V,

then $mgh = \frac{1}{2}mV^2$

With this velocity V, when body passes through the sand floor it comes to rest after traveling a distance x. Let F be the resisting force acting on the body. Net force in downward direction = $Mg - F$

We know that work done by all the forces is equal to change in KE : $(Mg - F)x = 0 - \frac{1}{2}MV^2$

or $(MG - F)\,x = -Mgh$ or $Fx = Mgh + Mgx$

or $F = Mg\left(1 + \frac{h}{x}\right)$

39.

In presence of friction, both the spring force and the frictional act so as to oppose the compression of the spring.

Work done by the net force

$$W = -\frac{1}{2}kx_m^2 - \mu mgx_m$$

Where x_m is the maximum compression of the spring. Change in kinetic energy

$$\Delta K = K_f - K_i = 0 - \frac{1}{2}mv^2$$

According to work-energy theorem

$$W = \Delta K$$

$$-\frac{1}{2}kx_m^2 - \mu mgx_m = -\frac{1}{2}mv^2$$

$$kx_m^2 + \mu mgx_m = \frac{1}{2}mv^2$$

$$kx_m^2 + 2\mu mgx_m - mv^2 = 0$$

$$x_m^2 + \frac{2\mu mgx_m}{k} - \frac{mv^2}{k} = 0$$

It is a quadratic equation in x_m.

Solving this quadratic equation for x_m and taking only the positive root since x_m is positive, we get

$$x_m = -\frac{\mu mg}{k} + \frac{1}{k}\sqrt{(\mu mg)^2 + mkv^2}$$

40.

Coefficient of restitution is defined as

$$e = \sqrt{\frac{H}{h}}$$

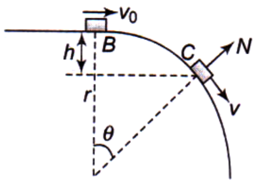
where h is the drop height and H is the bounce height.

So, $\frac{H}{h} = e^2 \Rightarrow H = e^2 h$.

41..

Equate loss of gravitational potential energy with gain of elastic potential energy.

42.



At point C: $mg \cos\theta - N = \frac{mv^2}{r}$

(when block leaves the surface normal force becomes zero, so putting $N = 0$) $\Rightarrow g\cos\theta = \frac{v^2}{r}$

$$v^2 = v_0^2 + 2gh,\ h = r - r\cos\theta$$

$$gr\cos\theta = (0.5\sqrt{gr})^2 + 2g(r - r\cos\theta)$$

$$\Rightarrow \quad \cos\theta = \frac{1}{4} + 2 - 2\cos\theta \Rightarrow \cos\theta = \frac{3}{4}$$

43.

Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

The ball will swing in a circle of radius $R = (L - d)$ about the peg. If the ball is to travel in the circle, the minimum centripetal acceleration at the top of the circle must be that of gravity:

$$\frac{mv^2}{R} = mg \Rightarrow v^2 = g(L - d)$$

When the ball is released from rest, $U_i = mgL$, and when it is at the top of the circle, $U_i = mg2(L - d)$, where height is measured from the bottom of the swing. By energy conservation,

$$mgL = mg2(L - d) + \frac{1}{2}mv^2$$

From this and the condition on v^2 we find $d = \frac{3L}{5}$.

44.
45.

$$mgl = \frac{1}{2}mu^2 \Rightarrow u^2 = 2g \qquad \text{(i)}$$

$$v^2 = u^2 + 2as \Rightarrow 0 = 2g - 2a(3)$$

$$\Rightarrow \quad a = \frac{g}{3} \Rightarrow \mu_k g = a$$

$$\therefore \quad \mu_k g = \frac{g}{3} \quad \therefore \mu_k = \frac{1}{3}$$